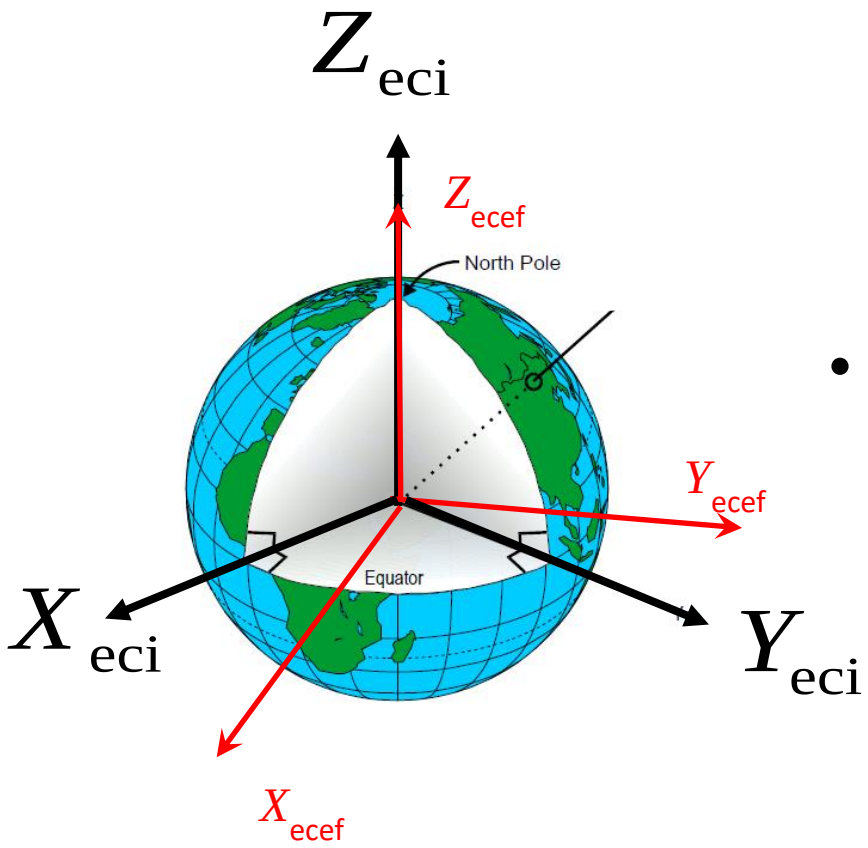
AAE 3520
Flight Vehicle Dynamics
Fall 2024

Angular velocity in ECI and NED frames

Learning Objectives

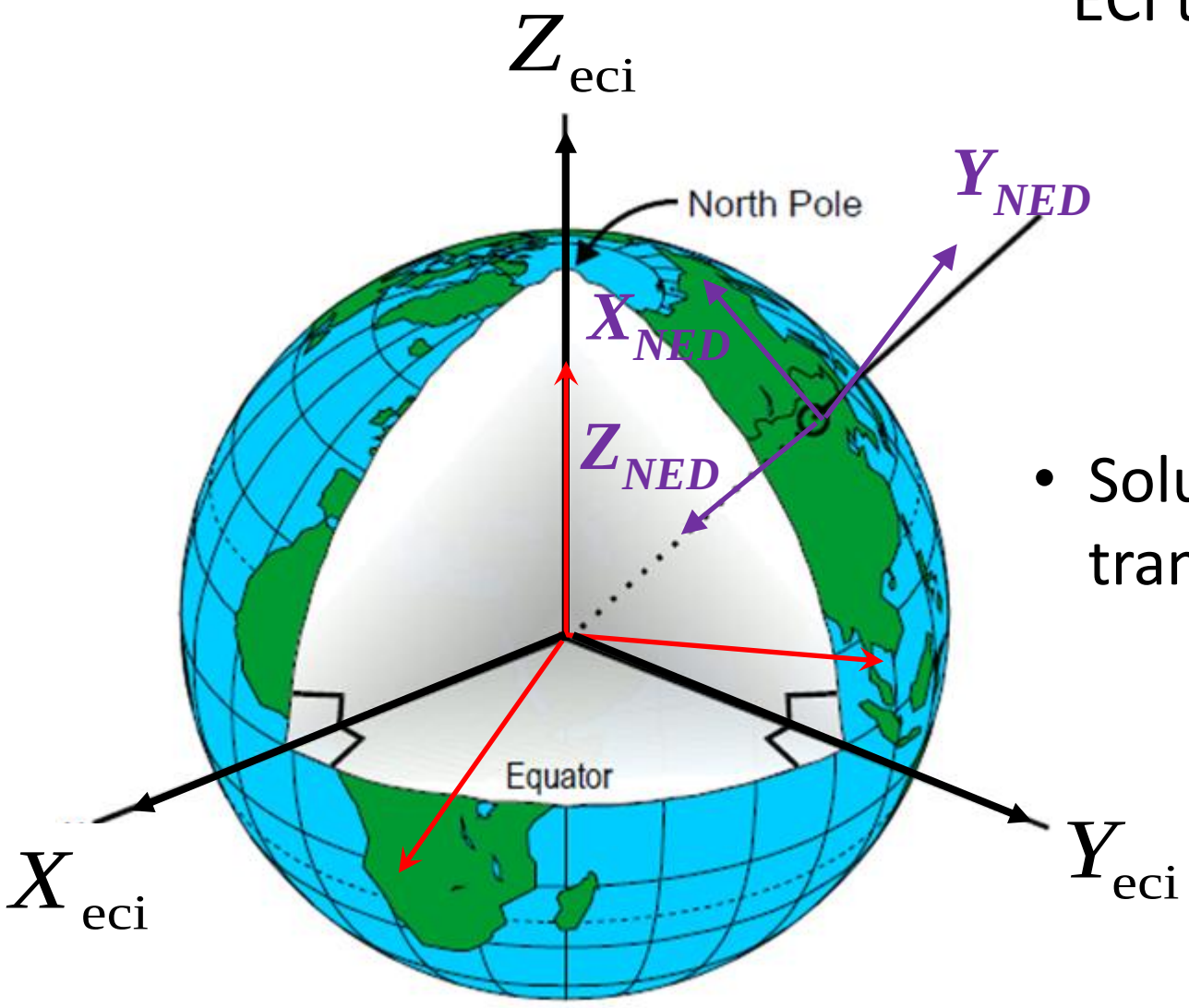
- Determine Euler angles between ECI and NED
- Compute relative velocity in both

Problem



- **ECI frame is fixed in the inertial space**
  - X: in the equatorial plane, pointing to vernal equinox
  - Z: points north along the earth rotational axis
  - Y: in the equatorial plane, completing a RHC system
- **ECEF:**
  - X: in the equatorial plane, at Greenwich (London longitude)
  - Z: similar to ECI Z-axis
  - Y: in the equatorial plane, completing a RHC system
- ECEF completes one rotation about ECI every 24 hours

Problem



- Transform vectors in NED to ECI and from ECI to NED

- Solution: compute the DCM transformation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ECI} = [DCM] \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NED}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NED} = [DCM]^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ECI}$$

DCM construction algorithm

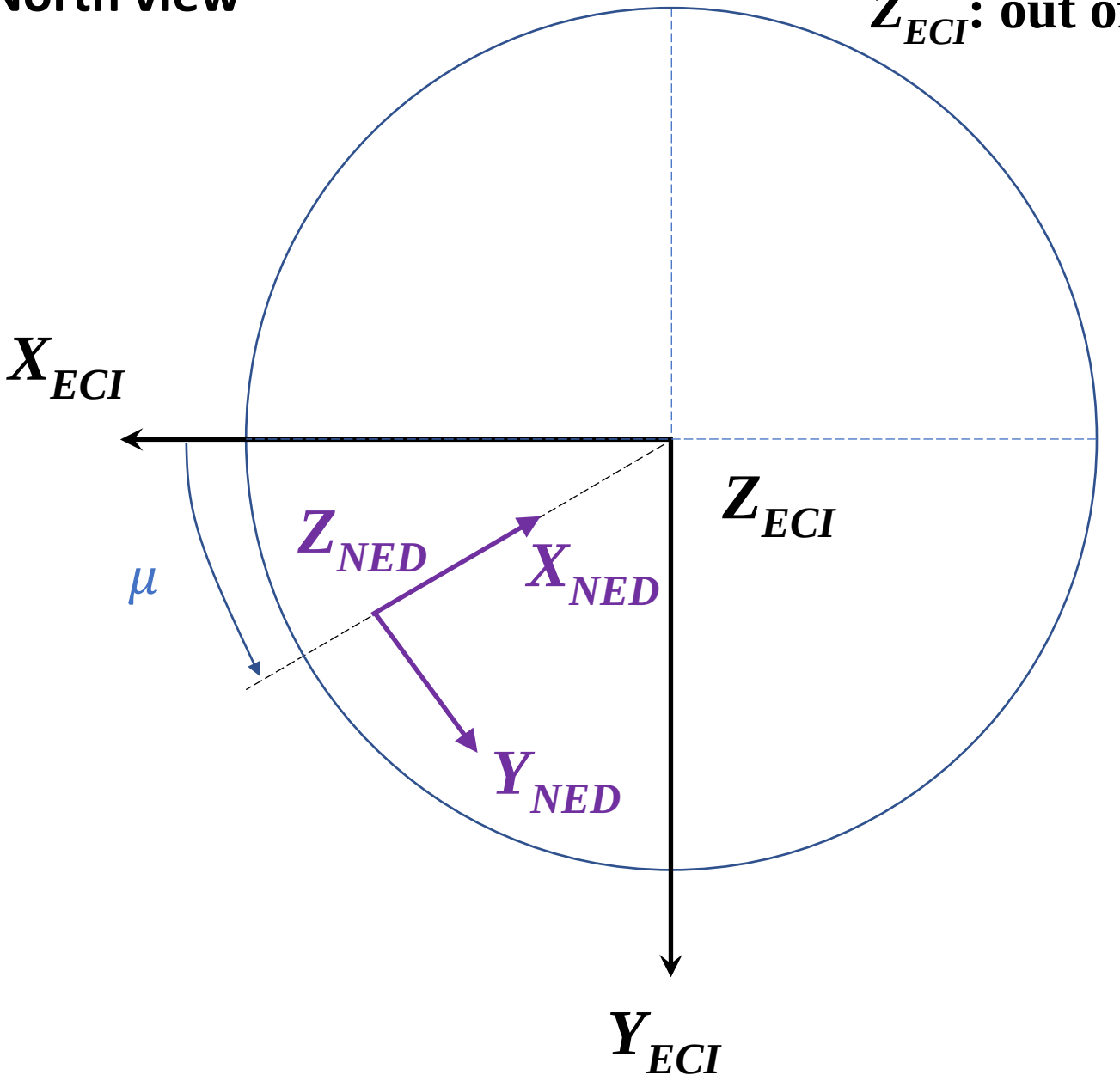
To find the DCM transformation, we need to find the Euler angles that transforms the ECI frame into the NED frame. These normally can be achieved by making 2 rotations:

1. A Rotation μ (longitude) about the Z-ECI axis. This operation places the X'-ECI axis at the NED longitude
2. Rotate the intermediate frame a -(90+ λ) about the Y'-ECI, this will align prime frame with the NED frame.

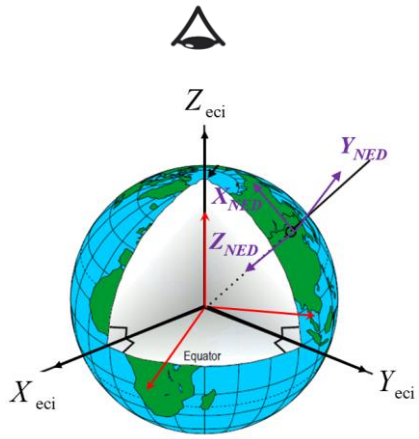
North view

Z_{NED}: hidden under X_{NED}

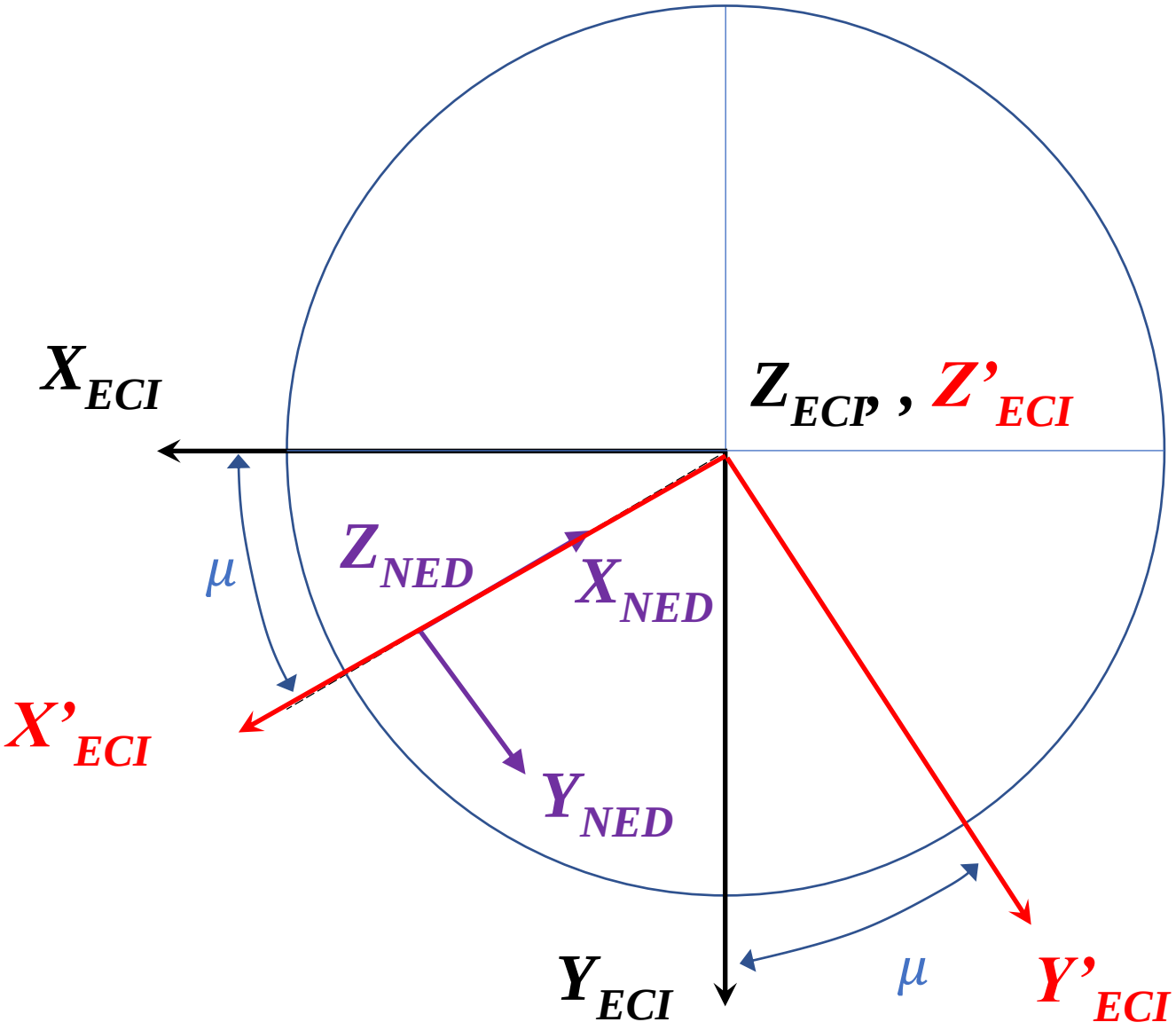
Z_{ECI}: out of the board



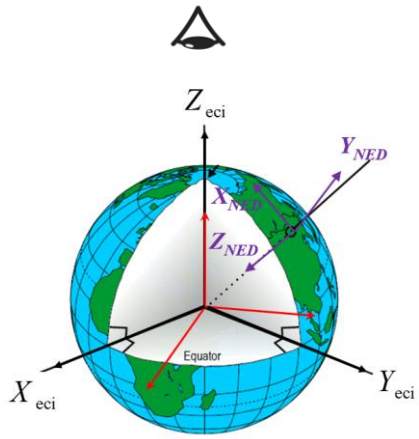
North view



1: First: a rotation μ about Z_{ECI}



North view



$\mu = \psi$

$-Z'_{ECI}$ **is the same as** Z_{ECI}

$-X'_{ECI}$, Y'_{ECI} **rotate** μ **angle about** Z_{ECI}

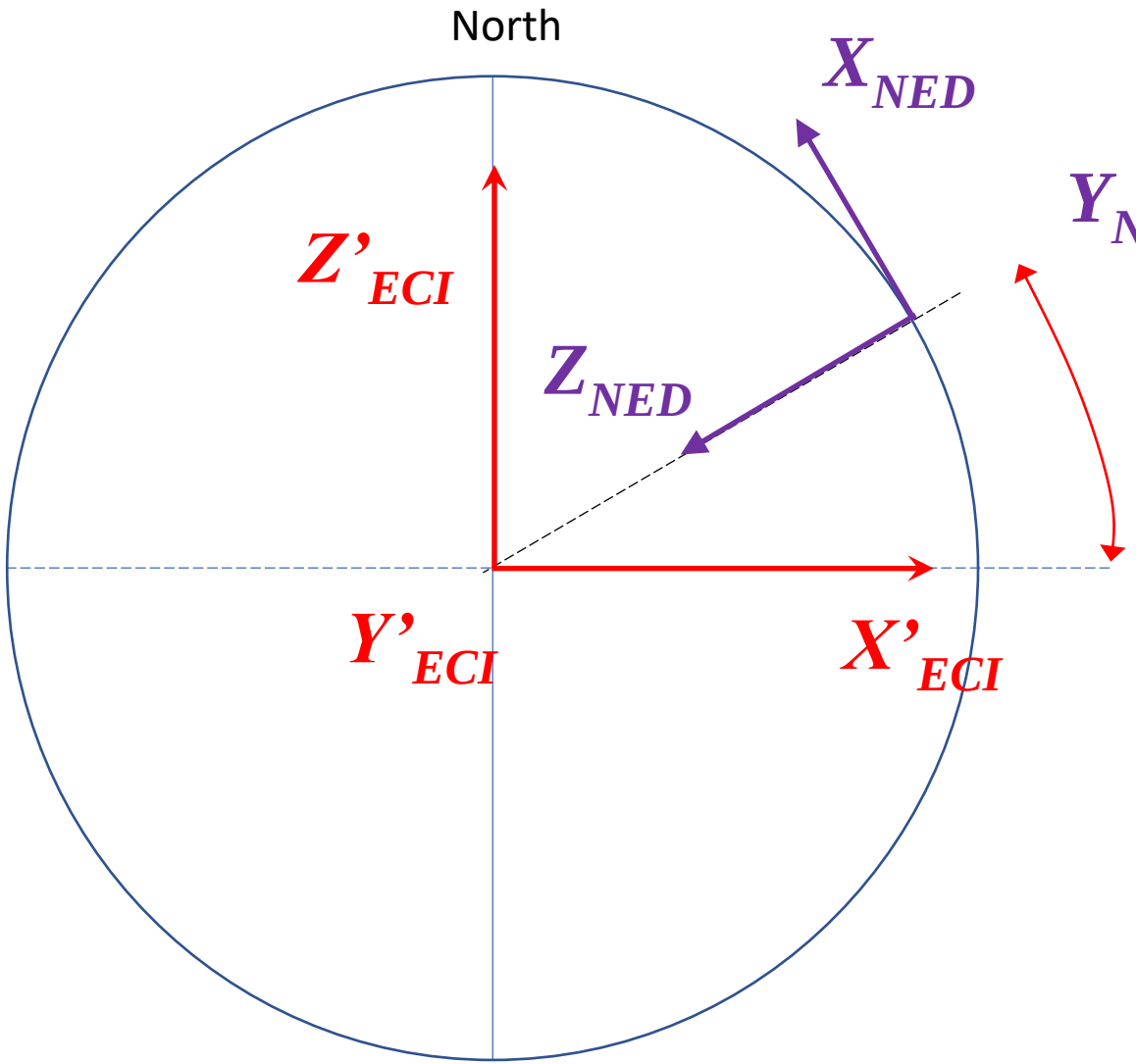
2: Second: a rotation $-(90+\lambda)$ about Y'_{ECI}

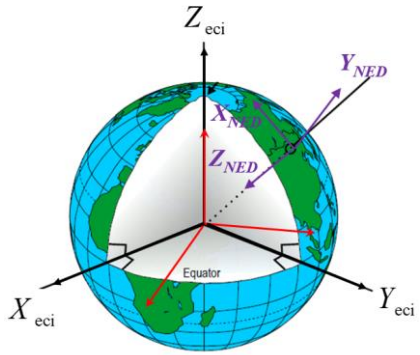
View: Perpendicular to the X'_{ECI} - Z'_{ECI} plane

Y'_{ECI}: into the board

Y_{NED}: into the board

Before



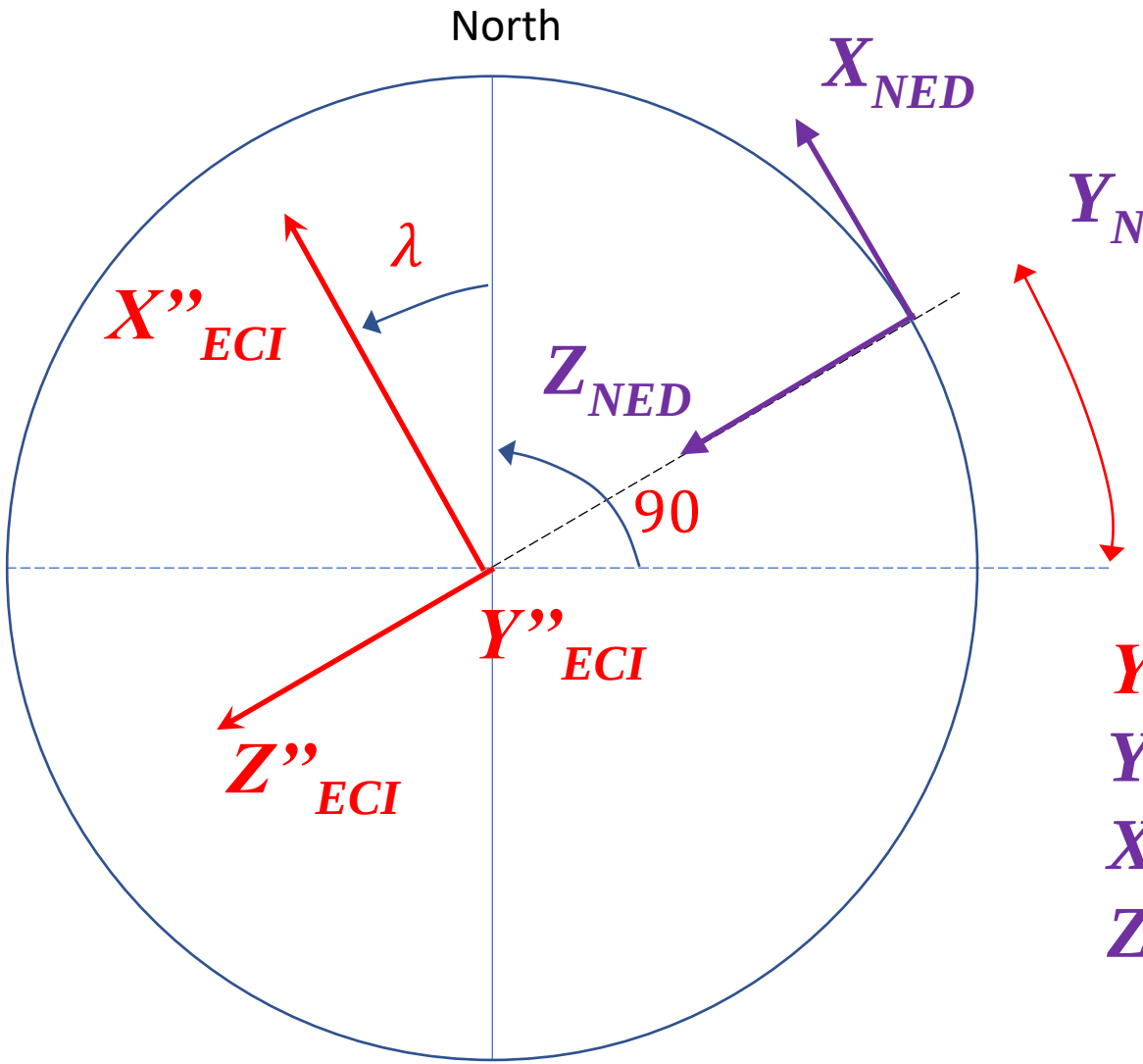


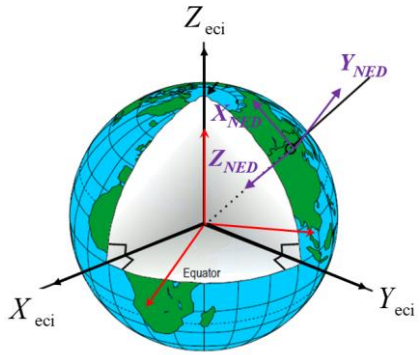
$$-(90+\lambda)=\theta$$

2: Second: a rotation $-(90+\lambda)$ about Y'_{ECI}

View: Perpendicular to the X'_{ECI} - Z'_{ECI} plane

After





Y''_{ECI}: into the board

Y_{NED}: into the board

X_{NED} and X''_{ECI} are aligned

Z_{NED} and Z''_{ECI} are aligned

DCM can be computed with: $\begin{cases} \phi = 0 \\ \boldsymbol{\theta} = -(\mathbf{90} + \boldsymbol{\lambda}) \\ \boldsymbol{\psi} = \mu \end{cases}$

$$DCM = \begin{bmatrix} \cos\Psi\cos\Theta & \cos\Psi\sin\Theta\sin\Phi - \sin\Psi\cos\Phi & \cos\Psi\sin\Theta\cos\Phi + \sin\Psi\sin\Phi \\ \sin\Psi\cos\Theta & \sin\Psi\sin\Theta\sin\Phi + \cos\Psi\cos\Phi & \sin\Psi\sin\Theta\cos\Phi - \cos\Psi\sin\Phi \\ -\sin\Theta & \cos\Theta\sin\Phi & \cos\Theta\cos\Phi \end{bmatrix}$$

$$DCM^{-1} = \begin{bmatrix} \cos\Psi\cos\Theta & \sin\Psi\cos\Theta & -\sin\Theta \\ \cos\Psi\sin\Theta\sin\Phi - \sin\Psi\cos\Phi & \sin\Psi\sin\Theta\sin\Phi + \cos\Psi\cos\Phi & \cos\Theta\sin\Phi \\ \cos\Psi\sin\Theta\cos\Phi + \sin\Psi\sin\Phi & \sin\Psi\sin\Theta\cos\Phi - \cos\Psi\sin\Phi & \cos\Theta\cos\Phi \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ECI} = [DCM] \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NED}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NED} = [DCM]^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ECI}$$

END